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(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601 Roll No.

B.Tech

(SEM I) ODD SEMESTER THEORY EXAMINATION 2009-10

MATHEMATICS-I

Time : 3 Hours] *[Total Marks : 100*

SECTION - A

All parts of this question are compulsory 2×10=20

1 (a) If $f(x) = f(0) + k\, f_1(0) + \frac{k^2}{L^2} f_2(\theta k),\ 0 < \theta < 1$ then the value of θ when $k = 1$ and $f(x) = (1-x)^{5/2}$ is given as ________

(b) The shortest distance from the point (1, 2, -1) to the sphere $x^2 + y^2 + z^2 = 24$ shall be ________.

(c) The Jacobian $J\left(\frac{u,\ v}{x,\ y}\right)$ for $u = e^x \sin y$, $v = x \log \sin y$ shall be ________.

(d) For the curve $ay^2 = x^2(a-x)$, which of the following statement(s) is/are Incorrect ?

(i) Curve passes through origin

(ii) Curve is symmetrical about y axis

(iii) Curve has two branches

(iv) Curve has no tangents at origin.

(e) If P and Q are non-singular matrices, then for Matrix 'M', which of the following statements are correct ?

(i) **Rank** $(PMQ) >$ **Rank** M

(ii) **Rank** $(PMQ) =$ **Rank** M

(iii) **Rank** $(PMQ) <$ **Rank** M

(iv) **Rank** $(PMQ) =$ **Rank** M + **Rank** (PQ)

(f) If λ is an eigen value of the matrix 'M' then for the matrix $(M - \lambda I)$, which of the following statement(s) is/are correct ?

(i) Skew symmetric

(ii) Non singular

(iii) Singular

(iv) None of these.

Indicate **True / False** for the following statements :

(g) For $\int_0^\infty \int_x^\infty f(x\,y)\,dx\ dy$, the change of order of integration is

(i) $\int_0^\infty \int_0^\infty f(x\,y)\,dx\ dy$ True / False

(ii) $\int_x^\infty \int_0^\infty f(x\,y)\,dx\ dy$ True / False

(iii) $\int_0^\infty \int_0^y f(x\,y)\,dx\ dy$ True / False

(iv) $\int_0^\infty \int_0^x f(x\,y)\,dx\ dy$ True / False

(h) The value of $\left|\overline{-\frac{1}{2}}\right.$ is given by

(i) $\sqrt{\pi}$ True / False

(ii) $2\sqrt{\pi}$ True / False

(iii) $-\sqrt{\pi}$ True / False

(iv) $-2\sqrt{\pi}$ True / False

Pick up the correct option from the following :

(i) If $\vec{F}$ is the velocity of a fluid particle then $\int_C \vec{F} \cdot d\bar{r}$ represents

(i) Work done

(ii) Circulation

(iii) Flux

(iv) Conservative field.

(j) The value of $\iint_S \vec{F}\cdot\vec{n}\; d\bar{s}$ where

$\vec{F} = ax\hat{i} + by\hat{j} + cz\hat{k}$; a, b, c being constants is given by

(i) $\frac{\pi}{3}(a+b+c)$

(ii) $\frac{4\pi}{3}(a+b+c)$

(iii) $2\pi(a+b+c)$

(iv) $\pi(a+b+c)$.

SECTION - B

Attempt any **three** parts of the following : **10×3=30**

2 (a) Determine the values of 'a' and 'b' for which the following system of equations has

$3x+5y-az=7$,

$x-by+4z=-3$,

$ax+4y-5z=4$

(i) No solution

(ii) A unique solution

(iii) Infinite no. of solutions.

(b) Find the value of $D^n\{x^{n-1}\log x\}$, $D^n \equiv \frac{d^n}{dx^n}$.

(c) If $u = \frac{(x+y)}{z}$, $v = \frac{(y+z)}{x}$, $w = \frac{y(x+y+z)}{(xz)}$ then show that u, v, w are not independent and find the relation between them.

(d) A rigid body is rotating with constant angular velocity w about a fixed axis. If 'v' is the linear velocity of any point of the body then prove that curl $v = 2w$.

(e) Assuming $\lceil n \lceil 1-n = \pi \operatorname{cosec} n\pi$, $0 < n < 1$, show that $\int_0^{\infty} \frac{x^{p-1}}{1+x}dx = \left(\frac{\pi}{\sin p\pi}\right)$; $0 < p < 1$.

SECTION - C

All questions of this section are compulsory. Attempt any **two** parts from each question : **10×5=50**

3 (a) If $x = \sin\left(\frac{\log y}{a}\right)$ then evaluate the value

$$(1-x^2)y_{n+2}-(2n+1)x\,y_{n+1}-(n^2+a^2)y_n=0$$

with usual symbols.

(b) If $u = \cos^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0.$$

(c) Verify Euler's theorem for $z = \dfrac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}$.

4 (a) In a plane ΔABC, find the maximum value of $\cos A \cos B \cos C$.

(b) If $x = e^{v} \sec u$, $y = e^{v} \tan u$ then evaluate $\dfrac{\partial(x, y)}{\partial(u, v)}$.

(c) The power 'P' required to propel a steamer of length 'l' at a speed 'u' is given by $P = \lambda u^3 l^3$ where λ is constant. If u is increased by 3% and l is decreased by 1%, find the corresponding increase in 'P'.

5 (a) Show that row vectors of the matrix

$$\begin{bmatrix} 1 & 2 & -2 \\ 1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$ are linearly independent.

(b) Find the rank of the following matrix using the elementary transformations $\begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 1 & -2 \end{bmatrix}$.

(c) Express the matrix $A \begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -i & 2-i & 2+i \end{bmatrix}$ as a sum of Hermitian and Skew Hermitian matrix.

6 (a) Interpret the physical meaning of $curl\ \vec{F}$ and $div\ \vec{F}$.

(b) Verify the divergence theorem for the function $\vec{F} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\,\hat{k}$; taken over the cube bounded by planes $x = 0$, $x = 1$; $y = 0$, $y = 1$; $z = 0$, $z = 1$.

(c) If a vector field is given by $\vec{F} = (x^2 - y^2 + x)\,\hat{i} - (2xy + y)\,\hat{j}$. Is this field irrotational ? If so, find its scalar potential.

7 (a) Evaluate $\iint_R \left(1 - \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}\right) dx\, dy$ over the first quadrant of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$!

(b) Find the mass of the region bounded by ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$; if the density varies as the square of the distance from the centre.

(c) A triangular prism is formed by the planes whose equations are $ay = bx$, $y = 0$ and $x = a$. Find the volume of this prism between the plane $z = 0$ and the surface $z = c + xy$.